Tailored free-form optics with movement to integrate tracking in concentrating photovoltaics

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Abstract: The economic use of high-efficiency solar cells in photovoltaics requires high concentration of sunlight and therefore precise dual-axis tracking of the sun. Due to their size and bulkiness, these trackers are less adequate for small- to mid-scale installations like flat rooftops. Our approach to combine concentrating and tracking of sunlight utilizes two laterally moving lens arrays. The presented analytic optics design method allows direct calculation of the free-form lens surfaces while incorporating the lateral movement. The obtained concentration performance exceeds a factor of 500. This demonstrates that one can benefit from high-efficiency solar cells and more compact and flexible single-axis trackers at the same time.

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References and links
1. Introduction

Concentrating photovoltaic (CPV) systems employ optics to concentrate direct sunlight onto small solar cells. The significant decrease of the required solar cell area provides a pathway to lower cost, as expensive semiconductor material is replaced with inexpensive mirrors or lenses by a factor roughly equal to the concentration factor. Furthermore, high-efficiency multi-junction (MJ) solar cells made by III-V elements can be used to boost the conversion efficiency of CPV modules beyond 30%, making CPV the most efficient among the PV technologies. To be economically viable, the use of expensive MJ solar cells needs high concentration - typically exceeding 400 suns [1]. For such high levels of concentration, CPV systems convert mainly the direct solar irradiation (DNI) and normally demands an accurate dual-axis tracking of the sun’s diurnal and seasonal movement. Most of today’s CPV manufacturers in the high-concentration range tend to work with very large modules and pedestal-mounted dual-axis trackers as illustrated by three examples in Fig. 1.

Well-suited for utility-scale power plants, these installations are less adequate for small to mid-scale solutions with very limited availability of land. Single-axis trackers, on the other hand, are already in use on flat rooftops for photovoltaic (PV) modules. However, CPV systems designed for single-axis trackers are limited in the achievable level of concentration to about $300 \times$ for polar alignment, where the tracker axis equals the earth’s axis of rotation [2]. This value may not be high enough to make economic use of MJ solar cells.
Despite the vast number of differing CPV system designs, almost all have something essential in common: the CPV modules - consisting of stationary concentrating optics plus deployed solar cells - and the external trackers are treated separately. In contrast to this clear separation, the concept of tracking integration embeds part of the external solar tracking functionality in the CPV module [3]. This offers a strong reduction of the external solar tracking effort whilst maintaining high concentration levels. Tracking integration thus enables to benefit from high-efficiency solar cells and more compact and flexible single-axis trackers at the same time: opening new small to mid-scale installation markets for high concentrating photovoltaics.

This article is structured as follows. The general concept, current developments and our approach of tracking integration in concentrating photovoltaics are discussed in Sec. 2. Analysis of the optical design problem establishes the link in Sec. 3 with previously published work on a new analytic optics design method [4, 5]. This insight is key to an analytic description that incorporates the lateral movement of the optics and allows to calculate the free-form lens surfaces directly. The concentration performance of the calculated free-form lenses is evaluated in Sec. 4, demonstrating that it is possible to benefit from high-efficiency solar cells and more compact and flexible single-axis trackers at the same time. Finally, in Sec. 5, conclusions are drawn and an outlook is given.

## 2. Tracking integration in concentrating photovoltaics

The concept of tracking integration in CPV is receiving more and more attention. Very different optical designs following this approach have been discussed in literature, and emerging start-up companies (see, for example [6–8]) underline the level of interest in this new concept.

### 2.1. General classification

In general terms, it is possible to distinguish between two specific application types.

1. Reduce the external tracking effort in favour of a compact installation, for example [9–14]

2. Fine-tune the tracking functionality allowing coarse external solar tracking [15]

Furthermore, it is of course possible to combine both functionalities to reduce the external tracking effort and to allow coarse remaining external solar tracking at the same time. For both types, it is essential to introduce additional degrees of freedom to the CPV module, such as the relative movement between optics and solar cells. Relative movements can be achieved through rotary motion or rectilinear motion as it is illustrated in Figs. 2(a) and 2(b). Figure 2(c) shows a motion-free alternative. The tracking integration is based on liquid crystal (LC) light steering for line concentration. These are only some examples of the numerous application possibilities for tracking integration in CPV.

![Fig. 2. Examples of different tracking integration concepts: using (a) rotary or (b) rectilinear motion; (c) a motion-free example based on liquid crystal (LC) light steering.](image-url)
On the other hand, additional degrees of freedom raise the complexity of the CPV module and more optics arrays will reduce the optical efficiency. A detailed benefit-cost analysis will be necessary to identify the most promising concepts.

Depending on the application type and the chosen optical design for tracking integration, external solar tracking might still be needed. A clear drawback of reducing the external solar tracking is the reduced yearly insolation (incoming solar radiation) due to off-axis cosine losses, especially in case of statically mounted solar panels. For dual-axis trackers, as illustrated in Fig. 3(a), the yearly insolation is maximal as the modules always point at the sun. However, the comparison of the annual insolation and the potential energy yield for polar aligned single-axis tracker with dual-axis tracker PV systems shows moderate differences, e.g. for miscellaneous places in Europe [16]. Figure 3(b) shows such a polar aligned single-axis tracker where the tracker axis equals the earth’s axis of rotation. For horizontal single-axis trackers, as shown in Fig. 3(c), the yearly insolation is typically lower.

![Fig. 3. Shown are different solar trackers used in both PV and CPV; (a) dual-axis tracker, (b) polar aligned single-axis tracker, and (c) horizontal single-axis tracker.](image)

It can be concluded that the annual energy yield alone does not justify the use of dual-axis trackers for any PV systems. Their use in conventional high concentrating photovoltaics is required due to the narrow acceptance angle of the optics. Tracking integration allows to overcome this limitation and to use high-efficiency multi-junction solar cells on single-axis trackers.

2.2. Tracking integration using laterally moving optics

In our approach, tracking integration is used to further increase the point concentration ratio for polar aligned single-axis tracker installations. The polar alignment offers a strong reduction of the external solar tracking effort in comparison to dual-axis tracking whilst maintaining high insolation levels. The laterally moving optics arrays combine both concentration and steering of the incident direct sun light. Figure 4 illustrates the inherent difference between conventional CPV and tracking integration. Figure 4(a) shows a conventional CPV module - consisting of stationary concentrating optics plus deployed solar cells - and the external dual-axis tracker. Figure 4(b) shows the schematic assembly of our module for an external single-axis tracker. Figure 4(c) shows the lens design that is used in this work. It is based on two plan-convex lenses with rectangular apertures. Both lenses can be moved laterally in x-direction to track the sun, as it is indicated by arrows in the figure. The restriction to lateral movement has the advantage that it keeps the CPV module thin, compared to the case where vertical movement is allowed as well. The polar aligned single-axis tracker is installed towards South. In this particular configuration, the tilt angle is equal to the latitude of the installation site and the rotational axis of the single-axis tracker equals the earth’s axis of rotation. The aperture angle which has to be covered by the tracking integration equals $\pm 24^\circ$ (axial tilt of the earth) in North-South direction.
Fig. 4. (a) Schematic assembly of a conventional CPV module for pedestal-mounted dual-axis trackers. (b) Our tracking integrated CPV module for polar aligned single-axis trackers. (c) The laterally moving lens design in this paper is based on two plan-convex lenses with rectangular apertures, suitable for lens array structures.

3. Derivation of the tailored free-form solution

The optical design in Fig. 4(c) is based on an extended version [17] of the Simultaneous Multiple Surface (SMS) design method in three dimensions [18], proprietary technology of LPI (www.lpi-llc.com). It allows the simultaneous calculation of two optical surfaces focusing two off-axis ray sets. This design has two distinct features: its wide field of view to cover ±24° and a clear separation between the two optical surfaces.

In a previously published paper [4], we demonstrated that a single lens that is characterised by these distinct features can couple three ray sets with only two lens surfaces. Such a lens is shown in Fig. 5(a). These two lens profiles can be calculated with a new analytic design method in two dimensions, presented in that paper. As one main result in [4], we demonstrated that this analytic design can be fully described by two variables only, the slope values \( m_0 \) and \( m_1 \) at the convergence points. The convergence points construction is shown in Fig. 5(b). They are characterized by on- and off-axis rays sharing identical points and normal vectors on each lens profile, and are key in our design method. Convergence points were first introduced in the design of aspheric V-groove reflectors [19,20]. The evaluation of the solution space we presented in [21], revealed the wide range of possible lens surface shapes that can be calculated with this design method. Later, we generalized the analytic optics design method to three dimensions [5].

To get from the analytic design of a single thick lens to the tracking integration optics design shown in Figs. 5(c) and 5(d), two extensions of the analytic design method are necessary.

Fig. 5. (a) The 2D analytic design of the single thick lens is based on the convergence points construction in (b). (c) This analytic lens design can be extended for tracking integration by including separated plano-convex lenses and (d) a shifted second lens for off-axis rays.
First, the single lens has to be "divided" into two plano-convex lenses to allow relative movement between the two optical surfaces. In the second step, the lateral movement of the plano-convex lenses is included in the same way we proposed in [3]. This means that the lower lens is shifted by an off-set $x_0$ in $x$-direction for the off-axis design angle. The following analytic description of the tracking integration optics design problem starts from the convergence points.

### 3.1. Analytic calculation starting from convergence points

Figure 6(a) shows the convergence points construction for the two plano-convex lenses. Due to the overall symmetry of this design, the initial ray construction lies within the sectional plane $y = 0$ of the lenses and can be used for both the 2D and 3D mathematical description.

The first convergence point coordinates $(\pm x_0, 0, z_0)$ (plus-minus sign because of $y$-$z$-plane mirror symmetry) on the upper lens surface can be freely chosen without loss of generality. Due to the lens’ mirror symmetry with respect to the $x$-$z$-plane, the associated normal vector is completely described by the single variable $m_0$; the slope in $x$-direction. The on-axis ray refracted at $(x_0, 0, z_0)$ intersects the first planar surface at $(p_{2x0}, 0, p_0)$ and the second planar surface at $(p_{3x0}, 0, q_0)$, before it intersects with the off-axis ray passing through the convergence point $(-x_0, 0, z_0)$ and the two planar surfaces at $(q_{2x0}, 0, p_0)$ and $(q_{3x0}, 0, q_0)$. The intersection determines the point coordinates $(x_1, 0, z_1)$ of the second convergence point. The lateral off-set $x_0$ of the second lens for the off-axis ray is taken into account by shifting the point coordinates to $(x_1 + x_0, 0, z_1)$. The normal vector at the second convergence point is also described by a single slope variable $m_1$. Further refractions at the second convergence point result in the focus positions $(0, 0, d)$ for the on-axis and $(r, 0, d)$ for the off-axis ray. For a chosen off-axis design angle, the initial ray construction can be fully described by the two slope values $m_0$ and $m_1$, the lenses’ refractive indices $n_2$ and $n_3$, and the longitudinal positioning of the planar lens surfaces described by their $z$-coordinates $p_0$ and $q_0$.

Next, all necessary surface functions describing the optical system have to be introduced. Therefore, two explicit surface functions $z = f(x, y)$ and $z = g(x, y)$ are defined to describe the curved lens surfaces. Additional mapping functions are introduced to completely describe the optical paths of rays passing through the optical system. Figure 6(c) shows an on-axis ray passing through an arbitrary point $p_1 = (x, y, f(x, y))$ on the first surface, mapped on $p_2 = (p_{2x}(x, y), p_{2y}(x, y), p_0)$ and $p_3 = (p_{3x}(x, y), p_{3y}(x, y), q_0)$, and then refracted in $p_4 = (s(x, y), t(x, y), g(s(x, y), t(x, y)))$ towards the focal point $p_5 = (0, 0, d)$. Figure 6(d) shows the identical construction for an arbitrary off-axis ray passing through the optical system along the
functions must be infinitely differentiable and have a power-series representation in two

\[ q_1 = (-x, y, f(x, y)), \ q_2 = (q_{2x}(x, y), q_{2y}(x, y), p_0), \ q_3 = (q_{3x}(x, y), q_{3y}(x, y), q_0) \]

and

\[ q_4 = (u(x, y), v(x, y), g(u(x, y), v(x, y))) \]

towards the focal point \( q_5 = (r, 0, d) \). The total optical path length of a ray can be expressed as the sum of sections using vector geometry as

\[ d_1 = v_0 \cdot (\vec{p}_1 - \vec{w}_0), \ d_2 = n_2|\vec{p}_2 - \vec{p}_1|, \ d_3 = |\vec{p}_3 - \vec{p}_2|, \ d_4 = n_3|\vec{p}_4 - \vec{p}_3|, \ d_5 = |\vec{p}_5 - \vec{p}_4| \]

(1)

for on-axis rays, and as

\[ \hat{d}_1 = \hat{v}_1 \cdot (\hat{q}_1 - \hat{w}_0), \ \hat{d}_2 = n_2|\hat{q}_2 - \hat{q}_1|, \ \hat{d}_3 = |\hat{q}_3 - \hat{q}_2|, \ \hat{d}_4 = n_3|\hat{q}_4 - \hat{q}_3|, \ \hat{d}_5 = |\hat{q}_5 - \hat{q}_4| \]

(2)

for off-axis rays. The vectors \( \vec{v}_0 \) and \( \vec{v}_1 \) denote the directional vectors for on- and off-axis ray sets, respectively. The position vector \( \vec{w}_0 \) denotes an arbitrary but fixed point on both plane wave-fronts and \( n_2 \) and \( n_3 \) denote the refractive indices of the first and second lens at the design wavelength.

Fermat’s principle is applied to derive all necessary functional differential equations to calculate the analytic solution. These equations are given as

\[ D_1 = \frac{\partial}{\partial x}(d_1 + d_2) = 0, \quad D_2 = \frac{\partial}{\partial y}(d_1 + d_2) = 0, \quad D_3 = \frac{\partial}{\partial p_{2x}}(d_2 + d_3) = 0, \]
\[ D_4 = \frac{\partial}{\partial p_{2y}}(d_2 + d_3) = 0, \quad D_5 = \frac{\partial}{\partial p_{3x}}(d_3 + d_4) = 0, \quad D_6 = \frac{\partial}{\partial p_{3y}}(d_3 + d_4) = 0, \]
\[ D_7 = \frac{\partial}{\partial s}(d_4 + d_5) = 0, \quad D_8 = \frac{\partial}{\partial t}(d_4 + d_5) = 0 \]

(3)

for on-axis, and as

\[ D_0 = \frac{\partial}{\partial x}(\hat{d}_1 + \hat{d}_2) = 0, \quad D_{10} = \frac{\partial}{\partial y}(\hat{d}_1 + \hat{d}_2) = 0, \quad D_{11} = \frac{\partial}{\partial q_{2x}}(\hat{d}_2 + \hat{d}_3) = 0, \]
\[ D_{12} = \frac{\partial}{\partial q_{2y}}(\hat{d}_2 + \hat{d}_3) = 0, \quad D_{13} = \frac{\partial}{\partial q_{3x}}(\hat{d}_3 + \hat{d}_4) = 0, \quad D_{14} = \frac{\partial}{\partial q_{3y}}(\hat{d}_3 + \hat{d}_4) = 0, \]
\[ D_{15} = \frac{\partial}{\partial u}(\hat{d}_4 + \hat{d}_5) = 0, \quad D_{16} = \frac{\partial}{\partial v}(\hat{d}_4 + \hat{d}_5) = 0 \]

(4)

for off-axis rays. The lens design as introduced in Fig. 6 is then fully described by 16 functional differential Eqs. (3) and (4) for fourteen unknown functions \( f(x, y), g(x, y), p_{2x}(x, y), p_{2y}(x, y), p_{3x}(x, y), p_{3y}(x, y), q_{2x}(x, y), q_{2y}(x, y), q_{3x}(x, y), q_{3y}(x, y), s(x, y), t(x, y), u(x, y) \) and \( v(x, y) \). Supposing that there exists an analytic and smooth solution \( (f, g, p_{2x}, p_{2y}, p_{3x}, p_{3y}, q_{2x}, q_{2y}, q_{3x}, q_{3y}, s, t, u, v) \) to the functional differential Eqs. (3) and (4), Taylor’s theorem implies that the functions must be infinitely differentiable and have a power-series representation in two variables. Thus the two lens surface functions and the twelve mapping functions can be given by power-series

\[ f(x, y) = \sum_{i=0}^{m} \sum_{j=0}^{m} f_{i,j}(x - x_0)^{j} y^{2j}, \quad g(x, y) = \sum_{i=0}^{m} \sum_{j=0}^{m} g_{i,j}(x - x_1)^{j} y^{2j} \]

(5)

\[ s(x, y) = \sum_{i=0}^{m} \sum_{j=0}^{m} s_{i,j}(x - x_0)^{j} y^{2j}, \quad t(x, y) = \sum_{i=0}^{m} \sum_{j=1}^{m} t_{i,j}(x - x_0)^{j} y^{2j-1} \]

(6)

with \( s(x, y) = (p_{2x}, p_{3x}, q_{2x}, q_{3x}, s, u)(x, y) \) and \( t(x, y) = (p_{2y}, p_{3y}, q_{2y}, q_{3y}, t, v)(x, y) \)

(7)

centred at convergence points \((x_0, 0, z_0)\) and \((x_1, 0, z_1)\), respectively. The exponents for \( y \)-coordinate take already into account that all functions are either even (provided by \(2j\)) or odd...
can be calculated by solving eight equations. The overall solution for arbitrary order solutions have confirmed the validity of this scheme. The in Figs. 6(a) and 6(b) introduced and in Eqs. (5)-(7) assigned initial conditions then satisfy the conditional equations $D_i = 0$ for $i = 1...16$ and provide general solutions for the initial Taylor series coefficients depending upon variables $m_0, m_1, p_0$ and $q_0$.

To our knowledge, the existence and uniqueness of solutions to similar systems of functional differential equations have neither been discussed in detail nor proven up to now. However, we have identified the following solution scheme that seems to provide both exact and unique solutions. So far, all evaluated ray tracing results for different explicitly calculated analytic solutions have confirmed the validity of this scheme. The overall solution for arbitrary order can be calculated by solving eight equations

$$\lim_{x \to x_0, y \to 0} \frac{\partial^n}{\partial x^n} \frac{\partial^m}{\partial y^m} D_i = 0 \quad (i = 1, 3, .., 15), \quad \{n \in \mathbb{N}_1, m = 0\}$$

for $x$-coordinate dependency, eight equations

$$\lim_{x \to x_0, y \to 0} \frac{\partial^n}{\partial x^n} \frac{\partial^m}{\partial y^m} D_i = 0 \quad (i = 2, 4, .., 16), \quad \{n = 0, m \in \mathbb{N}|\text{odd number } m\}$$

for $y$-coordinate dependency, and fourteen equations

$$\lim_{x \to x_0, y \to 0} \frac{\partial^n}{\partial x^n} \frac{\partial^m}{\partial y^m} D_i = 0 \quad (i = 2, 4, .., 16), \quad \{n \in \mathbb{N}_1, m \in \mathbb{N}|\text{odd number } m\}$$

$$\lim_{x \to x_0, y \to 0} \frac{\partial^{n-1}}{\partial x^{n-1}} \frac{\partial^{m+1}}{\partial y^{m+1}} D_i = 0 \quad (i = 3, 5, 7, 11, 13, 15), \quad \{n \in \mathbb{N}_1, m \in \mathbb{N}|\text{odd number } m\}$$

for mixed terms with $x$-$y$-coordinate dependency. To derive the solution scheme, it is useful to introduce an ordinal number $o = m + n$. There are two cases needed to be solved:

1. For $o = 1$, the sets of Eqs. (8) and (9) each result in eight non-linear algebraic equations for Taylor series coefficients $f_{2,0}, g_{2,0}, P_{2x1,0}, P_{3x1,0}, q_{2x1,0}, q_{3x1,0}, s_{1,0}, t_{1,0}$, and for $f_{0,2}, g_{0,2}, P_{2x0,1}, P_{3x0,1}, q_{2x0,1}, q_{3x0,1}, t_{0,1}, v_{0,1}$, respectively. These equations have each two general solutions, where one solution can be discarded as non-physical. The remaining two unique solutions can be expressed as functions of the initial, already known Taylor coefficients.

2. For $o \geq 2$, the sets of Eqs. (8), (9) and (10) result in three systems of linear equations for particular Taylor series coefficients which can be expressed as compact matrix equations, each with the same number of equations and unknowns

$$M_x \begin{pmatrix} f_{n+1,0} \\ g_{n+1,0} \\ s_{n,0} \end{pmatrix} = \vec{b}^{(n,0)}, \quad M_y \begin{pmatrix} f_{0,m+1} \\ g_{0,m+1} \\ t_{0,m} \end{pmatrix} = \vec{b}^{(0,m)}, \quad M_{xy} \begin{pmatrix} f_{n,m+1} \\ g_{n,m+1} \\ s_{n-1,m+1} \\ t_{n,m} \end{pmatrix} = \vec{b}^{(n,m)}$$

for $x$-coordinate dependency, $y$-coordinate dependency and $x$-$y$-coordinate dependency, respectively. The vector elements $s$ and $t$ are a compact notation of all mapping functions as defined in (7) for $x$- and $y$-direction, respectively. The matrix elements of $M_x, M_y$ and $M_{xy}$ consist of mathematical expressions which depend on Taylor series coefficients calculated for the initial conditions.
The needed vector elements of $\vec{b}^{(n,m)}$ are mathematical expressions only dependent on previously calculated Taylor series coefficients for $o = 2, 3, \ldots, (n + m - 1)$ and can be calculated for each ordinal number $o = 2, 3, \ldots$ in ascending order and for all possible combinations of $n$ and $m$ from equations (8)-(10). For known matrices $M_x, M_y$, and $M_{xy}$ and vectors $\vec{b}^{(n,m)}$, the Taylor series coefficients can be calculated by solving the linear systems of Eqs. (11).

The presented solution provides a scheme to calculate the Taylor polynomial coefficients in Eqs. (5)-(7) up to an arbitrary but finite order. The presented calculations are fully implemented in Wolfram Mathematica.

4. Optical system implementation and evaluation of concentration performance

All mathematical expressions are calculated up to $o = 15$, exported as C++ code and compiled in a MATLAB mex file. This library returns 81 calculated Taylor polynomial coefficients for each lens surface $f(x, y)$ and $g(x, y)$, 72 Taylor polynomial coefficients for each mapping function $s(x, y)$ and 64 Taylor polynomial coefficients for each mapping function $t(x, y)$ for any given (physically meaningful) initial parameters $(\theta, m_0, m_1, p_0, q_0, x_s, n_2)$.

The intended focusing functionality of the optical system suggests to choose a negative value for slope $m_0$. Local solutions in the neighbourhood of the convergence points exist for a wide range of different parameter sets. However, the lenses’ overall smoothness and symmetry additionally requires $\partial_x f(x, y) = \partial_x g(x, y) = 0$ for $x = 0$. This boundary condition introduces a complex correlation between the initial parameters, meaning that for a specific value $m_0$ the remaining parameters have to be chosen accordingly to meet the boundary condition. The analytic design method does not guarantee that the surfaces are differentiable in $y$-$z$-plane ($x = 0$). At this time, we cannot provide any mathematical proof but only state that the so far calculated free-form surfaces were sufficiently smooth.

Figures 7(a) and 7(b) show an explicit cross-sectional solution in two dimensions for $\theta = \pm 20^\circ$. Both the on-axis ray set and off-axis ray set for the shifted second lens are perfectly focused. Figures 7(c) and 7(d) show the equivalent free-form solution in three dimensions.

![Fig. 7](image)

Fig. 7. Ray tracing results for explicitly calculated solutions in two and three dimensions demonstrate perfect coupling of on- and off-axis ray sets, as intended by the analytic design.

With the successful implementation, the concentration performance can now be evaluated to demonstrate the suitability of this design approach for our tracking integration concept and to compare its concentration performance with previously designed optics.
4.1. Concentration performance evaluation of the analytic solution

Our initial optics design proposed in [3] was based on an extended Simultaneous Multiple Surface (SMS) algorithm in two dimensions to design rotational symmetric laterally moving plano-convex lenses. However, circular lens apertures are not suitable for application in lens array structures. In an additional paper [17], the design method has been generalized based on the Simultaneous Multiple Surface algorithm in three dimensions (SMS3D) [18], capable to design free-form lens surfaces with rectangular apertures. Figure 8 shows the plotted concentration ratio against the incident angle for rudimentary optimized rotational symmetric (SMS2D) and free-form (SMS3D) lens designs, analysed for monochromatic light (refractive index $n_2 = 1.5$).

Due to the overall symmetry of the designs, only half of the $\pm 24^\circ$ field of view needs to be evaluated. As both designs are based on the SMS design method, they each show one concentration ratio peak at the design angle.

To ensure comparability with these results, the concentration performance evaluation of the new analytic solution is done analogously. This means, the point concentration ratio is investigated for monochromatic light at refractive indices $n_2 = n_3 = 1.5$, a $\pm 24^\circ$ field of view and for $\pm 0.28^\circ$ sun’s divergence angle. The actual receiver size was defined in such a way that 95% of the energy entering the first lens aperture was collected at the square receiver (matching typical solar cells’ aspect ratio). The initial design parameters ($\theta = 21.4, m_0 \approx -0.24, m_1 \approx 0.23, p_0 \approx 0.44, q_0 \approx 0.62, x_s \approx 0.81$) have been determined by basic optimization to ensure good overall performance over the entire field of view. For this explicitly calculated solution, the slope values $\partial_x f(x,y)$ for $x = 0$ increases from $2 \times 10^{-4}$ at the origin to almost $5 \times 10^{-2}$ for the maximum value in $y$-direction. The slope values $\partial_y g(x,y)$ increases from $5 \times 10^{-16}$ at the center, to $3 \times 10^{-3}$ at the edge of the lens in $y$-direction. The ray tracing simulation in this work is done using the MATLAB-based ray tracer OPS (by courtesy of Prof. Dr. Udo Rohlffing, Hochschule Darmstadt, Germany). As OPS does not provide ray tracing for polynomial surface functions, $f(x,y)$ and $g(x,y)$ are approximated by $B$-spline curves of 3rd order defined on a grid. This surface approximation also ensures the differentiability of the two surfaces in $y$-$x$-plane.

The optimal off-set for the second lens and the receiver is in all cases obtained by the ray tracer. For angles up to the respective design angle the mapping for both, second lens and receiver position is close to a law $r(\theta) \propto \tan(\theta)$. For this final design, the total motion range is 0.94 for the first lens, and 0.37 for the second lens, given in relative units referenced to the lens side length in $x$-direction. The analogous evaluation of an analytic solution based on

![Figure 8](image_url)
the here presented new design method is added to the same graph. As it turned out, the best concentration performance has been obtained for a 2:1 aspect ratio ($x:y$ direction) for both the plano-convex lenses and the solar cell receiver (to use two identical squared solar cells). The correspondent curve in Figure 8 shows the concentration performance as a function of the incident angle without any additional optimization of the shapes of the analytically calculated surfaces. As expected from the analytic design approach, there are two concentration ratio peaks. One at the design angle $21.4 ^\circ$ similar to the SMS3D design, plus one additional peak at $0 ^\circ$. Apart from these distinct peaks, the overall concentration performance stays well above $500 \times$ over the entire field of view. The clear variation of the concentration performance results from the fact that the optics are designed to form a perfect image of the sun at the design angles. To ensure a more homogeneous illumination of the solar cells, the analytic solution can serve as an excellent starting point for further optimization of the optical system. If the full solar spectrum is considered, the concentration ratio will be considerably lower due to dispersion. Light of different wavelengths will have a different irradiance distribution and therefore affect the solar cell efficiency. However, this problem can be addressed by including a final stage concentrator on top of the solar cell. This way, further desired attributes of CPV systems, such as a uniform illumination of the cells [22] and good color mixing over the entire field of view could be achieved. A final polychromatic optimization will then help to tap the full potential of this tracking integration concept. A further extension could be the case in which the movement is no longer lateral but follows a curved trajectory. In case of beam-steering array optics designed with the SMS method, it has been shown recently that curved tracking trajectory helps to broaden the incident angle’s range significantly [23].

5. Conclusion

Within the scope of this work, we demonstrated how the concept of tracking integration in concentrating photovoltaics can play a key role to open small to mid-scale installation markets for CPV due to the strong reduction of the external solar tracking effort. With our system, based on two laterally moving lens arrays, it is possible to use more compact and flexible polar aligned single-axis trackers without being forced to abandon high-efficiency multi-junction solar cells.

To unleash the full potential of the considered CPV system, a free-form optics design is absolutely essential. We presented a new analytic optics design method capable of calculating the surfaces of our movable free-form lenses directly. Ray tracing results for the concentration performance confirm that the analytic moving design can serve as an excellent starting point for further optimization of the optical system including a final stage concentrator.

It is noteworthy to mention in this context, that the incorporation of movement in direct optics design methods could prove very useful for various other applications.

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